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Polarized parton distribution functions in the valon model framework, using QCD fits to Bernstein polynomials

Ali N. Khorramian[a,b*†], A. Mirjalili[b,c] and S. Atashbar Tehrani [b,d]

[a]Physics Department, Semnan University, Semnan, Iran

[b]Institute for Studies in Theoretical Physics and Mathematics (IPM), Tehran, Iran

[c]Physics Department, Yazd University, Yazd, Iran

[d]Physics Department, Persian Gulf University, Boushehr, Iran

In this paper polarized valon distribution is derived from unpolarized valon distribution. In driving polarized valon distribution some unknown parameters exist which must be determined by fitting to experimental data. Here we have used Bernstein polynomial method to fit QCD predictions for the moments of g_1^p structure function, to suitably the constructed appropriate average quantities of the E143 and SMC experimental data. After calculating polarized valon distributions and all parton distributions in a valon, polarized parton density in a proton are available. The results are used to evaluate the spin components of proton. It turns out that the results of polarized structure function are in good agreement with all available experimental data on g_1^p of proton.

1 INTRODUCTION

In valon model, the hadron is envisaged as a bound state of valence quark cluster called valon. For example, the bound state of proton consists of two U and one D valons. These valons thus bear the quantum numbers of respective valence quarks. Hwa [1] found the evidence of valons in the deep inelastic neutrino scattering data , suggested their existence and applied it to a variety of phenomena. Hwa and Yang published two papers [1] and improved the idea of valon model and extracted new results for the valon distributions. Recently we published a paper [2] in which we extracted unpolarized constituent quark and hadronic structure function in Next-to-Leading order. We were interested here to extend valon model to polarized ones which has not yet been done.

2 Unpolarized and Polarized Valon Distributions

Valence and its associated sea quarks plus gluons in the dressing processes of QCD is defined as *valon*. In a bound state problem these processes are virtual and a good approximation for the problem is to consider a valon as an integral unit whose internal structure cannot be resolved. The proton, for example, has three valons which, on the one hand, interacts with each other in a way that is characterized by the valon wave function and which on the other hand responds independently in an inclusive hard collision with a Q^2 dependence that can be calculated in QCD at high Q^2. This picture suggests that the structure function of a hadron involves a convolution of two distributions: valon distribution in proton and parton distributions in a valon. In an unpolarized situation we may write:

$$F_2^p(x,Q^2) = \sum_v \int_x^1 dy G_{v/p}(y) \mathcal{F}_2^v(\frac{x}{y}, Q^2) \ , \qquad (1)$$

the summation is over the three valons. Here $F_2^p(x,Q^2)$ is a proton structure function and $\mathcal{F}_2^v$ is the corresponding structure function of a v valon and $G_{v/p}(y)$ indicates the probability for the v valon to have momentum fraction y in the proton. In Ref. [1] the unpolarized valon distribution in proton with a new set of parameters for $G_{U/p}(y)$ and $G_{D/p}(y)$ has been recalculated. Now we can construct the polarized valon distributions in proton which are based on the definitions of unpolarized valon distributions. The polarized valon is defined to be a dressed polarized valence quark in QCD with the cloud of polarized gluons and sea quarks which can be resolved by high Q^2 probes. In [3] we found the following constrain for the polarized parton distributions, $|\delta f(x,Q^2)|$, and unpolarized one, $f(x,Q^2)$, at low value of Q^2 which by positivity requirements implying

$$|\delta f(x,Q^2)| \leq f(x,Q^2) \ ,$$

where $f = u, \overline{u}, d, \overline{d}, s, \overline{s}, g$, and furthermore by the sum rules

$$\Delta u + \Delta\overline{u} - \Delta d + \Delta\overline{d} = A_3 = 1.2573 \pm 0.0028 \ ,$$

*Speaker at the Workshop.

†khorramiana@theory.ipm.ac.ir

$$\Delta u + \Delta\overline{u} + \Delta d + \Delta\overline{d} - 2(\Delta s + \Delta\overline{s}) = A_8 = 0.579 \pm 0.025 ,$$

here Δf is the first ($n = 1$) moment of δf, which is defined by

$$\Delta f(Q^2) = \int_0^1 dx \delta f(x, Q^2) . \tag{2}$$

For determination polarized parton distributions, the fundamental analysis is to relate the polarized input density to unpolarized ones using some intuitive theoretical argument as guide lines. So we can introduce the following equations to relate the Non-singlet(NS) and Singlet(S) polarized valons to unpolarized valon distributions

$$\begin{aligned} \delta G_p^{NS}(y) &\equiv 2\delta G_{U/p}(y) + \delta G_{D/p}(y) \\ &= 2\delta F_U(y) \times G_{U/p}(y) + \delta F_D(y) \times G_{D/p}(y) , \end{aligned} \tag{3}$$

$$\begin{aligned} \delta G_p^{S}(y) &\equiv 2 f(y)\, \delta G_{U/p}(y) + f(y)\, \delta G_{D/p}(y) \\ &= 2 f(y)\, \delta F_U(y) \times G_{U/p}(y) + f(y)\, \delta F_D(y) \times G_{D/p}(y) , \end{aligned} \tag{4}$$

where $\delta F_j(y)$ and $f(y)$ are defined by:

$$\delta F_j(y) = N_j y^{\alpha_j} (1 - y)^{\beta_j} (1 + \gamma_j y + \eta_j y^{0.5}) , \tag{5}$$

$$f(y) = \kappa y^{0.5} + \lambda y + \mu y^{1.5} + \nu y^2 + \rho y^{2.5} + \tau y^3 . \tag{6}$$

the subscript j refer to U, D, and $G_{U/p}(y)$, $G_{D/p}(y)$ has been defined in Ref.[1]. The factor 2 in Eqs.(3,4) backs to existence of 2-U type valons. By using experimental data for g_1^p [4] and using Bernstein polynomials which we will explain them in Sec. 4, we did a fitting, and could get parameters of Eqs. (5,6) which are defined by unpolarized valon distributions U and D in Eqs. (3,4). According to the Eq. (2), we can write as an example, the first moment of polarized u-valence quark in valon model as following:

$$\Delta u_v = \int_0^1 dx \delta u_v = 2 \int_0^1 dy \delta G_{U/p}(y),$$

therefore we can construct for polarized valon distribution the following sum rules

$$2 \int_0^1 dy \delta G_{U/p}(y) - \int_0^1 dy \delta G_{D/p}(y) = A_3 ,$$

$$2 \int_0^1 dy \delta G_{U/p}(y) + \int_0^1 dy \delta G_{D/p}(y) = A_8 .$$

Here we have not considered the SU(3) symmetry breaking where with totally flavor-symmetric we have for light sea density $\delta\overline{u} = \delta\overline{d} = \delta\overline{s}$. So what we will get for A_3 and A_8 values will have a little bit different with respect to the quoted values in the above.

3 Moment of g_1^p polarized structure function

Let us define the Mellin moments of any structure function $h(x, Q^2)$ as following:

$$\mathcal{M}(n, Q^2) \equiv \int_0^1 x^{n-1} h(x, Q^2)\, dx . \tag{7}$$

Correspondingly in n-moment space we indicate the moments of polarized valon distributions for NS and S sector, $\Delta G_p^{NS}(n)$ and $\Delta G_p^{S}(n)$ as:

$$\Delta G_p^{NS,S}(n) = \int_0^1 y^{n-1} \delta G_p^{NS,S}(y) dy , \tag{8}$$

For the moments of polarized singlet and non singlet distributions we shall use, the leading order solutions of the renormalization group equation in QCD. They can be expressed entirely in terms of the evolution parameter s

$$s = \ln \frac{\ln Q^2/\Lambda^2}{\ln Q_0^2/\Lambda^2} , \tag{9}$$

where Q_0, Λ are scale parameters and we fixed it by $Q_0^2 = 1GeV^2$ and used $\Lambda = 0.203GeV$. It is completely known the moments of the unpolarized Singlet and Non-singlet and for polarized ones we have

$$\Delta M^{NS}(n, Q^2) = \exp(-\delta d_{NS}^{(0)n} s) , \tag{10}$$

$$\begin{aligned} \Delta M^{S}(n, Q^2) &= \frac{1}{2}(1 + \delta\rho) \exp(-\delta d_+ s) \\ &+ \frac{1}{2}(1 - \delta\rho) \exp(-\delta d_- s) , \end{aligned} \tag{11}$$

where $\delta\rho$ and other associated parameters are as follows:

$$\begin{aligned} \delta\rho &= \frac{\delta d_{NS}^{(0)n} - \delta d_{gg}^{(0)n} + 4 f \delta d_{qg}^{(0)n}}{\Delta} , \\ \Delta &= \delta d_+ - \delta d_- = \sqrt{(\delta d_{NS}^{(0)n} - \delta d_{gg}^{(0)n})^2 + 8 f \delta d_{qg}^{(0)n} \delta d_{gq}^{(0)n}} , \\ \delta d_\pm &= \frac{1}{2}(\delta d_{NS}^{(0)n} + \delta d_{gg}^{(0)n} \pm \Delta) , \\ b &= \frac{33 - 2f}{12\pi} , \end{aligned} \tag{12}$$

f is the number of active flavors. The anomalous dimensions $\delta d_{ij}^{(0)n}$ which are simply the n-th moment of polarized LO splitting function are given by [5]. The moments of all polarized u and d-valence quark in a proton are:

$$\Delta M_{u_v}(n, Q^2) + \Delta M_{d_v}(n, Q^2) = \Delta M^{NS}(n, Q^2) \times \Delta G_p^{NS}(n) \tag{13}$$

The moment of polarized singlet distribution in a proton (Σ) is as follows:

$$\Delta M_{\Sigma}(n, Q^2) = \Delta M^S(n, Q^2) \times \Delta G_p^S(n) \ . \tag{14}$$

In Eq.(14) Σ symbol indicates $\sum_{q=u,d,s}(q+\overline{q})$. In this situation we can compute the moments of polarized sea quarks as:

$$\Delta M_{\bar{q}}(n, Q^2) =$$

$$\frac{\Delta M_{\Sigma}(n, Q^2) - \Delta M_{u_v}(n, Q^2) - \Delta M_{d_v}(n, Q^2)}{2f} \ . \tag{15}$$

In moment space we can get the polarized moment of proton in terms of their polarized constituent quarks as below

$$\mathcal{M}(n, Q^2) = \frac{1}{2}\sum_{q=u,d,s}(e_q^2)(\Delta M_q + \Delta M_{\bar{q}}) \ . \tag{16}$$

By inserting Eqs. (13-15) in above equation, 16 unknown parameters will be appeared which should be determined by a fitting processes.

4 QCD fits to average of moments using Bernstein polynomials

Because for a given value of Q^2, only a limited number of experimental points, covering a partial range of values x, are available, one can not use the moments equation like the present one. A method device to deal to this situation is that to take averages of structure functions with Bernstein polynomials.
We define these polynomials as

$$p_{n,k}(x) = \frac{\Gamma(n+2)}{\Gamma(k+1)\Gamma(n+k+1)}x^k(1-x)^{n-k} \ , \tag{17}$$

Thus we can compare theoretical predictions with experimental results for the Bernstein averages, which are defined by [6]

$$g_{n,k}(Q^2) \equiv \int_0^1 dx p_{nk}(x) g_1(x, Q^2) \ , \tag{18}$$

In Eq. (17), $p_{n,k}(x)$ are normalized to unity, $\int_0^1 dx p_{n,k}(x) = 1$. Using the binomial expansion in Eq.(17), it follows that the averages of g_1 with $p_{n,k}(x)$ as weight functions, can be obtained in terms of odd and even moments,

$$g_{n,k} = \frac{(n-k)!\Gamma(n+2)}{\Gamma(k+1)\Gamma(n-k+1)}$$

$$\times \sum_{l=0}^{n-k} \frac{(-1)^l}{l!(n-k-l)!}\mathcal{M}((k+l)+1, Q^2) \ . \tag{19}$$

where moments $\mathcal{M}$ are given by

$$\mathcal{M}((k+l)+1, Q^2) = \int_0^1 x^{(k+l+1)-1} g_1(x, Q^2) dx \ , \tag{20}$$

The integral (18) represents an average of the function $g_1(x)$ in the region $\bar{x}_{n,k} - \frac{1}{2}\Delta x_{n,k} \leq x \leq \bar{x}_{n,k} + \frac{1}{2}\Delta x_{n,k}$ where $\bar{x}_{n,k}$ is the average of x which is very near to the maximum of $p_{n,k}(x)$, and $\Delta x_{n,k}$ is the spread of $\bar{x}_{n,k}$. The key point is that values of g_1 outside this interval contribute little to the integral (18), as $p_{n,k}(x)$ decreases to zero very quickly. So, by suitably choosing n, k, we manage to adjust the region where the average is peaked to that in which we have experimental data. This means that we can use only 41 averages $g_{n,k}$ like :

$$g_{2,1}^{(\exp)}(Q^2), g_{2,2}^{(\exp)}(Q^2), ..., g_{13,10}^{(\exp)}(Q^2).$$

Other restriction which we assume here, is that to ignore the effects of moments with high order n which are not very effective. To obtain these experimental averages from the E143 and SMC data for xg_1 [4], we fit $xg_1(x, Q^2)$ for each bin in Q^2 separately, to the convenient phenomenological expression

$$xg_1^{(phen)} = \mathcal{A}x^{\mathcal{B}}(1-x)^C \ , \tag{21}$$

this form ensures zero values for xg_1 at $x = 0$, and $x = 1$. Using Eq.(21) with the fitted values of $\mathcal{A}, \mathcal{B}, C$, one can then compute $g_{n,k}^{(exp)}(Q^2)$ using Eq.(18), in terms of Gamma functions. Using Eq.(19) the Bernstein averages $g_{n,k}(Q^2)$ can be written in terms of odd and even moments $\mathcal{M}(n, Q^2)$,

$$\begin{aligned}
&g_{2,1}(Q^2) = 6\left(\mathcal{M}(2, Q^2) - \mathcal{M}(3, Q^2)\right) \ , \\
&g_{2,2}(Q^2) = 3\left(\mathcal{M}(3, Q^2)\right) \ , \\
&\vdots \\
&g_{13,10}(Q^2) = 4004.000001\left(\mathcal{M}(11, Q^2) - \mathcal{M}(14, Q^2)\right) \ , \\
&-12012\left(\mathcal{M}(12, Q^2) - \mathcal{M}(13, Q^2)\right) \ .
\end{aligned} \tag{22}$$

We shall use the result of Eq.(21) for the QCD prediction of $\mathcal{M}(n, Q^2)$. Thus there are 16 parameters to be simultaneously fitted to the experimental $g_{n,k}(Q^2)$ averages. Defining a global χ^2 for all the experimental data points, we found an acceptable fit with minimum $\chi^2/\text{d.o.f.} = 0.03262/123$. Since polarized valon distribution are now determined with 16 *known* parameters, the Eqs. (5,6) are now completely determined for polarized NS and S valon distributions. In Fig.(1) we plotted $y \times \delta G_p^{NS}$ and $y \times \delta G_p^S$ as a function of y.

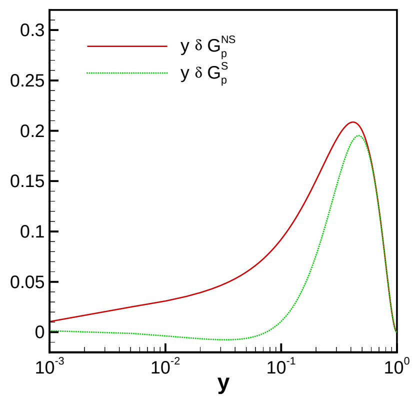

Figure 1. Non-singlet and Singlet polarized valon distributions as a function of y.

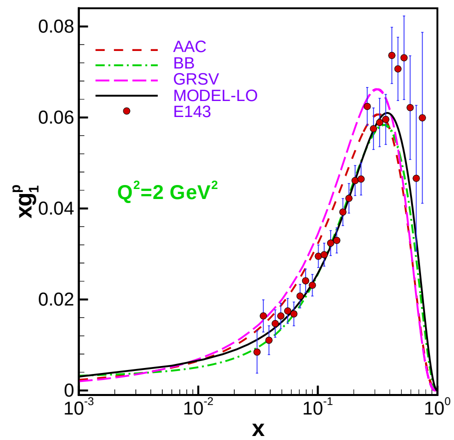

Figure 2. The results of $g_1^p(x, Q^2)$, at $Q^2 = 2\ GeV^2$. Our model is indicated by solid line. Other theoretical predictions are from Ref.[3,7]. Experimental data points are from Ref.[3]

5 Polarized parton distributions and g_1^p structure function

Now we want to compute the polarized structure function of proton. Since we calculated before polarized valon distribution in a proton, by having the polarized structure function in a valon, it is possible to extract polarized parton structure in a proton. To obtain the z dependence of parton distributions in practical purposes from the $n-$dependent exact analytical solutions in Mellin-moment space, one has to perform a numerical integral in order to invert the Mellin-transformation. Consequently we can get the following expressions for polarized parton distributions in a proton:

$$\begin{aligned}
\delta u_v(x, Q^2) &= 2\int_x^1 \delta f^{NS}(\frac{x}{y}, Q^2) \times \delta G_{U/p}(y)\frac{dy}{y} \ , \\
\delta d_v(x, Q^2) &= \int_x^1 \delta f^{NS}(\frac{x}{y}, Q^2) \times \delta G_{D/p}(y)\frac{dy}{y} \ , \\
\delta\Sigma(x, Q^2) &= \int_x^1 \delta f^{S}(\frac{x}{y}, Q^2) \times f(y)\ (2\delta G_{U/p}(y) \\
&\quad +\delta G_{D/p}(y))\frac{dy}{y} \ , \qquad (23)
\end{aligned}$$

In leading order approximation in QCD, according to the quark model, g_1^p can be written as a linear combination of δq and $\delta\overline{q}$,

$$g_1^p(x, Q^2) = \frac{1}{2}\sum_q e_q^2[\delta q(x, Q^2) + \delta\overline{q}(x, Q^2)] \ , \qquad (24)$$

where e_q are the electric charges of the (light) quark-flavors $q = u, d, s$. We are now in a position to present the results for the proton polarized structure function, g_1^p. We presented in Fig.(2) the results of $g_1^p(x, Q^2)$, for $Q^2 = 2\ GeV^2$ and compared it with experimental data [4].

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